Homework #3 (10 points) - Show all work on the following problems:

Problem 1 (2 points): Consider an electric field $\vec{E} = kr^3\hat{r}$ (with k a constant):

a. Find the charge density ρ as function of position.
b. Find the total charge contained in a sphere of radius R, by using Gauss's law.
c. Find the total charge contained in a sphere of radius R, by direct integration.

Problem 2 (1 point): Find the electric field as a function of radius r inside a sphere with uniform charge density ρ.

Problem 3 (1 point): Find the electric field as a function of radius r inside a sphere with charge density that increases linearly from the origin (ρ(r) = kr).

Problem 4 (2 points): Consider two partially overlapping spheres of radius R, carrying uniform but opposite charge densities ρ and -ρ. Show that the field in the region of overlap is constant, and express its value in terms of the vector $\vec{d}$ from the center of the positive sphere to the center of the negative sphere.

Problem 5 (2 points): One of the following vector functions is not a valid electrostatic field. By evaluating the curl, determine which one is impossible. For the one that is a valid electrostatic field, find the corresponding electric potential V.

a. $\vec{E} = xy\hat{x} + 2yz\hat{y} + 3xz\hat{z}$
b. $\vec{E} = y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z}$

Problem 6 (2 points): Find the electric potential V(r) as a function of radius inside and outside of a uniformly charged sphere with radius R and total charge Q, by integrating from infinity. Explicitly compute the gradient of this function and double-check that $-\nabla V$ gives the correct electric field inside and outside the sphere.